The Mathematical Parallels Between Packet Switching and Information Transmission

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Abstract: All communication networks comprise transmission systems and switching systems, even though they are usually treated as two separate issues. Communication channels are generally disturbed by noise from various sources. In circuit switched networks, reliable communication requires the error-tolerant transmission of bits over noisy channels. In packet switched networks, however, not only can bits be corrupted with noise, but resources along connection paths are also subject to contention. Thus, quality of service (QoS) is determined by buffer delays and packet losses. The theme of this paper is to show that transmission noise and packet contention actually have similar characteristics and can be tamed by comparable means to achieve reliable communication. The following analogies between transmission and switching are identified.

1. Buffering against contention is a process that is similar to the error correction of noise corrupted signals. A pseudo signal-to-noise ratio that represents the carried load of packet switches can be deduced from the Boltzmann model of packet distribution. We show that the maximal data rate of Clos network with random routing satisfies Shannon-Hartley's noisy channel capacity theorem.

2. When deflection routing is applied to the Clos networks the loss probability decreases exponentially, which is similar to the exponential behavior of the error probability of binary symmetric channels with random channel coding. In information theory, this result is stated as the noisy channel coding theorem.

3. The similarity between bipartite matching and expander graph manifests the resemblance between route assignments and error-correcting codes. An extension of Sipser and Spielman decoding algorithm of expander codes to route assignments of Benses networks is given to illustrate their correspondence.

4. Scheduling in packet switching serves the same function as source coding in digital transmission. The smoothness of scheduling, like noiseless channel coding, is bounded by entropy inequalities.

5. The sampling theorem of bandlimited signals provides the cornerstone of digital communication and signal processing. Recently, Birkhoff-von Neumann decomposition of traffic matrices has been widely applied to packet switches. With respect to the complexity reduction of packet switching, we show that the decomposition of a doubly stochastic traffic matrix plays a similar role to that of the sampling theorem in digital transmission. We conclude that packet switching systems are governed by mathematical laws that are similar to those of digital transmission systems as envisioned by Shannon in his seminal 1948 paper, A Mathematical Theory of Communication.

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Short Bio: Tony T. Lee received his BSEE degree from National Cheng Kung University, Taiwan in 1971, and his MS and PhD degrees in electrical engineering from Polytechnic University in New York, in 1976 and 1977, respectively. Currently, he is a Professor of Information Engineering at the Chinese University of Hong Kong, and an adjunct Professor of the Institute of Applied Mathematics of Chinese Academy of Science, Beijing. He has also held positions at the Polytechnic University of New York, AT&T Bell Laboratories, and Bellcore, currently Telcordia Technologies.