Abstract: Sparse linear arrays, such as co-prime arrays and nested arrays, have the attractive capability of providing up to $O(M^2)$ degrees of freedom using only $O(M)$ physical sensors. By exploiting their difference-coarray structure, an augmented sample covariance matrix can be constructed and the MUtiple SIgnal Classification (MUSIC) algorithm can be applied to identify more sources than the number of physical sensors. Our statistical performance analysis consists of two parts: (i) the derivation and analysis of the asymptotic (large sample) mean-squared error (MSE) of the MUSIC algorithm applied to the coarray model, namely, the coarray-based MUSIC algorithm; (ii) the derivation and analysis of the Cramér-Rao Bound (CRB). The augmented sample covariance matrix based on the coarray model can be constructed via either direct augmentation or spatial smoothing, leading to two coarray-based MUSIC algorithms, namely the direct augmentation based MUSIC (DA-MUSIC), and the spatial smoothing based MUSIC (SS-MUSIC). We derive the asymptotic MSE expression for both MUSIC algorithms, which is applicable even if the number of sources exceeds that of sensors. We show that DA-MUSIC and SS-MUSIC have the same asymptotic MSE. We also show that when there are more sources than the number of sensors, the asymptotic MSE converges to a positive value instead of zero when the signal-to-noise ratio (SNR) goes to infinity. This finding explains the "saturation" behavior of the coarray-based MUSIC algorithm in high SNR regions observed in the numerical examples of previous studies. The CRB gives a lower bound of the variance of unbiased estimators. We derive the CRB for general sparse linear arrays and analyze its behavior in high SNR regions. We show that when there are more sources than the number of sensors, the CRB shows unusual behavior by being strictly nonzero as the SNR goes to infinity. This behavior is consistent with our observation on the asymptotic MSE expression of DA-MUSIC and SS-MUSIC. We use numerical experiments to verify our analytical derivations, and demonstrate the application of our results to resolution analysis.