

# SEMINAR NOTICE

Preston M. Green Department of Electrical and Systems Engineering

## OPTIMAL CONTROL SYNTHESIS FOR INHOMOGENEOUS ENSEMBLES

PhD Preliminary Research Examination

**Anatoly Zlotnik**

PhD Candidate

Preston M. Green Department of Electrical and Systems Engineering  
Washington University in St. Louis

**Abstract:** An emerging area in mathematical control theory called Ensemble Control constitutes a class of problems that require the manipulation of an uncountably infinite collection of structurally identical or similar dynamical systems, which are indexed by a parameter set, by applying a common open-loop control. Investigation of this subject is motivated by compelling engineering problems in areas including quantum control, sensorless robotic manipulation, and neuroengineering, which involve ensembles of linear, bilinear, or nonlinear oscillating systems, for which analytical control laws are infeasible in practice or do not exist. While the controllability of many of these systems has been studied in detail, constructive control design methods for practical problems remain elusive.

The control that transfers an ensemble of finite-dimensional time-varying linear systems between states in function space satisfies a Fredholm integral equation of the first kind that characterizes the dynamics of the ensemble system. We introduce a computationally efficient, optimization-free algorithm based on the singular value decomposition (SVD) for the synthesis of minimum norm solutions to this equation, and show that the error in the terminal state of the ensemble vanishes as the discretization used becomes finer. The method has been applied to control ensembles of harmonic oscillators as well as more complex quantum transport systems. This technique is then extended to an iterative fixed-point method for optimization-free synthesis of ensemble controls for bilinear systems. We focus on bilinear systems evolving on the special orthogonal group  $SO(n)$ , and in particular on manipulating the Bloch system, which is fundamental to many phenomena in quantum control.

Beyond linear and bilinear systems, we examine the control of ensembles of nonlinear oscillators. This special class of nonlinear systems plays an important role in neuroscience, and the ability to optimally manipulate collections of such systems can improve the treatment of clinical disorders such as Parkinson's disease and epilepsy. We focus in particular on entrainment, which refers to the dynamic synchronization of an oscillating system to a periodic input. Phase coordinate transformation and formal averaging are employed to represent the asymptotic behavior of periodically forced nonlinear oscillators using autonomous scalar ordinary differential equations. The calculus of variations is then used to derive minimum energy and minimum time controls that entrain ensembles of non-interacting oscillators to a target frequency, or an integer multiple of such a target.

DATE: Thursday, February 28, 2013  
TIME: 10:10 a.m.  
PLACE: Green Hall, Room 0120

Thesis advisor:  
Dr. Jr-Shin Li

This seminar is in partial fulfillment  
of the Doctor of Philosophy degree