Function Approximation Methods
For Optimal Control Problems

by

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The optimal control problems of a dynamic system governed by ordinary differential equations arise in a wide range of applications such as astronautics, robotics, and economics. Up to now, enormous efforts have been spent on the development of practical computational techniques for solving optimal control problems. A traditional approach, which is know as the indirect method, is based on finding a solution that satisfies Pontryagin’s maximum principle or other necessary conditions. An alternative approach, the direct method, is to convert the optimal control problem in a function space to an optimization problem in a finite-dimensional space, which can be solved by nonlinear programming techniques.

The main topic of this dissertation is the development of a series of direct approaches based on function approximation methods for solving complicated nonlinear optimal control problems. Given the important advantages for the use of wavelets, we approximate both the state derivative and control functions by expansions in the Haar wavelet series with unknown coefficients. The differential and integral expressions which appear in the system dynamic and the objective functions are converted into algebraic equations in the unknown coefficients. In this way, the optimal control problem is replaced by an optimization problem. The epiconvergence of the approximating method is proven to ensure the approximating problems converge to the original problem. We then propose an iterative method, called the Sequential Quadratic-Quadratic Method, which contains a local quadratic model corresponding to minimizing a quadratic approximation of the objective function subject to quadratic approximations of the system differential equations. Lastly, we apply these results to the spacecraft attitude control problems and Fuller’s problem, and establish a connection between the multi-resolution analysis via wavelets and the multigrid method for future work.

DATE:       Wednesday, May 3, 2006
TIME:       11:00 a.m.
PLACE:       Bryan Hall, Room 305

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This seminar is in partial fulfillment of the Doctor of Science Degree