Clearing Payment Vector Modeling

By: Sam Jones, Mentor: Zachary Feinstein

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Abstract

This project examines how different strategies of selling stocks in a financial crisis effect both the health of a financial system and the firms within that system.

The systems we study are interconnected networks of firms in which each has financial liabilities to others. The value of each firm is often dependent on the payoffs they receive from their claims on other firms in the financial system.

To determine the health of the system and value of each firm, we find a clearing payment vector that efficiently clears the obligations of each firm. We first consider systems in which each firm has cash, but no additional assets, and systems in which each firm has cash and a variable amount of a specific stock.

We then consider systems more relevant to real-world settings in which each firm has cash and variable amounts of multiple stocks. In these systems, each firm has many possible strategies for selling stocks. Any given strategy can change the final value of the firm, which in turn affects the final value of connected firms and the overall health of the system.

We analyze the different strategies and apply them to different systems to determine which are best for the health of an individual firm and which are best for the health of the system.

The findings of this project can be used by individual firms as well as system regulators to determine the best strategies in a financial crisis.

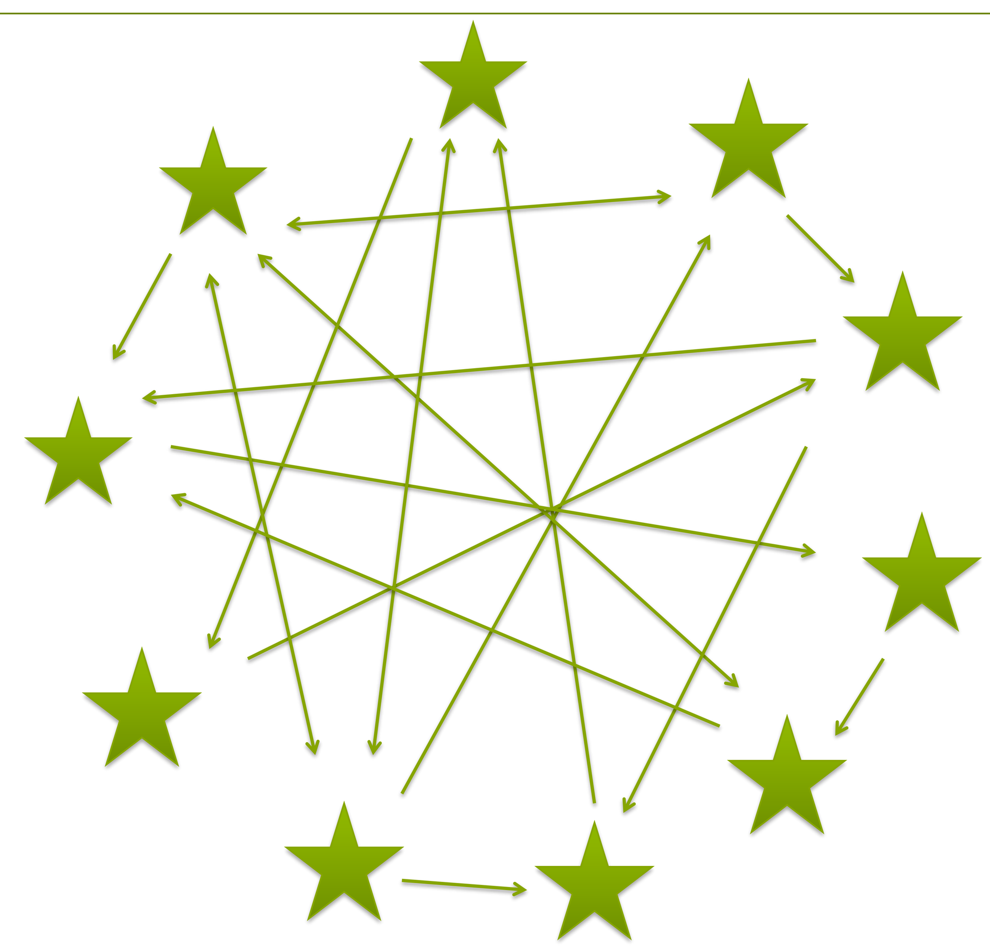

Figure 1. Example network of interconnected firms, arrows represent financial liabilities

Important Definitions

Clearing Payment Vector **p**: concave, increasing function of operating cash flow vector and level of nominal liabilities that clears the obligations of the members of the clearing system and represents a specification of the payments made by each of the nodes in the financial system.
Relative liabilities matrix **A**: nominal liability of one firm to another as a proportion of the debtor firm's total liabilities
Cash vector **x**: initial operating cash of each firm
Stock matrix **s**: initial amount of each firm's stocks
Total obligation vector $\bar{\mathbf{p}}$: payment level required for complete satisfaction of all contractual liabilities by all nodes
Stock market value $\mathbf{\Pi}$: equilibrium price of each stock

Clearing Payment Vector Equations

1. No Stock:

$$p(x) = \bar{p} \wedge (A^T p(x) + x)$$

2. One Stock:

$$p(x,s) = \bar{p} \wedge (A^T p(x,s) + x + \Pi(x,s)s)$$

$$\Pi(x,s) = F(\sum_{i=1}^{n} (\frac{(\bar{p}_i - x_i - \sum_{j=1}^{n} a_{ji} p(x,s))^+}{\Pi(x,s)} \wedge s_i))$$

3. Multiple Stocks:

$$p(x, s^1, ..., s^m) = \bar{p} \wedge (A^T p(x, s^1, ..., s^m) + x + \sum_{k=1}^{n} \Pi_k(x, s^1, ..., s^m) s^k))$$

a. General:

$$\Pi_{\hat{k}} = F_{\hat{k}}(\sum_{i=1}^{n} (\frac{(\bar{p}_i - x_i - \sum_{k \neq \hat{k}} (\Pi_k(x, s^1, ..., s^m) s_i^k) - \sum_{j=1}^{n} (a_{ji} p_j(x, s^1, ..., s^m)))^+}{\Pi_{\hat{k}}(x, s^1, ..., s^m)} \wedge s_i^{\hat{k}}))$$

b. Proportional:

$$\Pi_k = F_k(\sum_{i=1}^{n} ((\frac{(\bar{p}_i - x_i - \sum_{j=1}^{n} (a_{ji} p_j(x, s^1, ..., s^m)))^+}{\sum_{k=1}^{n} \Pi_k(x, s^1, ..., s^m) s_i^k} \wedge 1) s_i^k))$$

c. First Stock:

$$\Pi_k = F_k(\sum_{i=1}^{n} (\frac{(\bar{p}_i - x_i - \sum_{l=1}^{k-1} (\Pi_l(x, s^1, ..., s^m) s_i^l) - \sum_{j=1}^{n} (a_{ji} p_j(x, s^1, ..., s^m)))^+}{\Pi_k(x, s^1, ..., s^m)} \wedge s_i^k))$$

d. Most Stock:

$$\Pi_k = F_k(\sum_{i=1}^{n} (\frac{(\bar{p}_i - x_i - \sum_{l=j_{max(s^k)}}^{j_{min(s^k)}-1} (\Pi_l(x, s^1, ..., s^m) s_i^l) - \sum_{j=1}^{n} (a_{ji} p_j(x, s^1, ..., s^m)))^+}{\Pi_k(x, s^1, ..., s^m)} \wedge s_i^k))$$

e. Least Stock:

$$\Pi_k = F_k(\sum_{i=1}^{n} (\frac{(\bar{p}_i - x_i - \sum_{l=j_{min(s^k)}}^{j_{max(s^k)}-1} (\Pi_l(x, s^1, ..., s^m) s_i^l) - \sum_{j=1}^{n} (a_{ji} p_j(x, s^1, ..., s^m)))^+}{\Pi_k(x, s^1, ..., s^m)} \wedge s_i^k))$$

f. Medium Stock:

$$\Pi_k = F_k(\sum_{i=1}^{n} (\frac{(\bar{p}_i - x_i - \sum_{l=j_{median(s^k)}}^{j_{max(s^k)}-1, j_{min(s^k)}} (\Pi_l(x, s^1, ..., s^m) s_i^l) - \sum_{j=1}^{n} (a_{ji} p_j(x, s^1, ..., s^m)))^+}{\Pi_k(x, s^1, ..., s^m)} \wedge s_i^k))$$

Methods

The overall goal of this project is to use implement clearing payment vector formulas into MATLAB and use the resulting computer program to find the health of various financial systems. We can use the program to both compare different strategies of selling stock and compare different allocations of initial stock. In comparing clearing payment vectors of the same financial system, the most important factors to note are the overall health of the system and the health of each individual firm.

To find the health of an individual firm, we compare the clearing payment of that firm. More specifically, if the value of the clearing payment vector at point i of one strategy is greater than that of another strategy, the former strategy would make firm i healthier and lower its risk. Additionally, if the value of the clearing payment vector at point i of a certain strategy is lower than the value of the total obligation vector at point i, the firm would have defaulted in that strategy. If the values are equal, the firm would not have defaulted.

The simplest way to measure the overall health of the system is to take the sum of values of the clearing payment vector of each firm. If one strategy yields a sum value that is greater than another strategy, the former strategy would have a better impact on the health of the financial system and lower its risk. Also, it is important to note any changes in which firms default. Generally, if less firms default from a certain strategy, that strategy would make the financial system healthier.

Current & Future Work

The most important financial system which I have looked at is the case where each firm is given multiple stocks. Within this case, I am currently looking at how the different strategies of selling stock (generally, proportionally, etc.) effect the health of the system. By determining the most beneficial strategies to the system, we can make assumptions about how these strategies would affect firms in real-world financial systems.

In the future, I plan on also looking at more complex strategies. For example, I will start implementing mixed strategies in which different firms can sell off stocks using different methods. Also, I will continue to look at more varied initial conditions of the financial system. More specifically, I will continue to see how variations in the liabilities matrix, cash vector, and stock matrix change the effectiveness of different strategies.

Contact Information

Sam Jones
Washington University in St. Louis
Email: samtjones@wustl.edu
Phone: 7038643597

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