Feedback Control of Climate Dynamics

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1.0 Abstract

Climate change has become a topic of great importance in recent years, and interest in climate change has even reached the government level as policymakers are looking for ways to mitigate the effects of global warming. This project continues research started by Justin Ruths and Dr. Jr-Shin Li that tried to promote systems science as a tool for climate modeling. Our research consists of studying complex climate models with carbon feedbacks using control systems techniques. Formulation of an optimal control problem and solution via the pseudospectral method give insight into emissions scenarios needed to drive global mean surface temperature to specific values.

2.0 Background: Climate Modeling

Climate models quantitatively describe the evolution of some climatic characteristic over time. The underlying mathematics behind climate models are differential equations derived from the laws of physics, fluid dynamics, and chemistry. These models sometimes incorporate other equations that were fitted to be consistent with past data. Computer-based climate modeling began in the 1950s with numerical weather prediction models. Over time, climate modeling was expanded to many different applications, including atmospheric chemistry, climate variability, and global warming. ("Modeling Climate").

With recent concerns over global warming, climate change modeling now has importance in the political realm. Climate modeling is becoming an important tool to help policymakers draft plans to mitigate global warming. While climate models are not entirely accurate, they give policy-makers perspective on how our anthropogenic emissions affect the climate in both the near and long-term future. (U.S. Climate Change Science Program).

2.1 Types of Climate Models

Many different climate models exist. These models vary in many factors, including spatial and temporal resolution, complexity, underlying physical equations, and purpose. The following sections detail two types of models that are important in simulating global warming.

Global Circulation Models

One popular climate model used to simulate global warming is called a Global Circulation Model (GCM). Government research laboratories that employ these models include those at NASA and NOAA. These models simulate the movement of chemicals and energy throughout the atmosphere and oceans using the laws of chemistry, fluid dynamics, and physics. To employ these models, climate scientists discretize the area of interest into small boxes and describe the movement of materials and energy across the boundary of the boxes. The result is a large system of partial differential equations that can simulate how the atmosphere's temperature changes over time. These partial differential equations generally must be solved using algorithmic techniques, such as spectral methods or finite difference methods. The motivation for using these models is their high degree of detail and multidimensionality; they can have a resolution of 1-5 degrees in latitude/longitude and have varying levels of spatial resolution with regards to altitude. Unfortunately, the sheer size and complexity associated with a GCM generally requires supercomputing power. ("Global Climate Model).

Energy Balance Models

One of the first, but still widely used, type of climate model is the Energy Balance Model (EBM). These models evolve the Earth's surface temperature according to the difference in incoming and outgoing radiation (“Energy Balance Models”). According to the laws of thermodynamics, change in temperature is proportional to the absorbed radiation:

$$c\frac{dT}{dt} = R_{in} - R_{out}$$

where c is the specific heat capacity of the substance (the Earth). These models are usually zero-dimensional or one dimensional in latitude. The equations for incoming and outgoing radiation can have varying levels of detail account for various factors, such as reflectivity, atmospheric albedo, and atmospheric opacity. The benefit of using an EMB is that the low dimensionality and limited number of equations make it possible to run these models in a standard computing environment.

3.0 Purpose of Research

The interactions of chemicals and energy in the atmosphere are fairly complex; they involve several intricate equations with feedback of natural processes, such as the carbon cycle. The quantitative, complex nature of climate models seems to lend itself well to systems analysis. The majority of climate models, however, are missing a systems framework (Ruths, Li and Tung, “Global Climate Change”). In addition, most climate models are used to see how a specific emissions scenario could impact future climates. If a policymaker wanted to find an emissions profile that would drive the temperature to a specific point, they would have to use a shooting method and try several different scenarios before they hit the target.

The goal of this research is to approach climate change from a systems perspective. My research advisors, Justin Ruths and Dr. Jr-Shin Li, introduced a systems framework for the climate modeling problem by representing climate dynamics in a state space form. They also used optimal control and the pseudospectral method to find an emissions scenario that causes the global mean surface temperature to increase 2°C by 2050. This project continues their research and tries to accomplish the following tasks:

- Revisit the optimal control problem in Ruth’s and Li’s research using a different objective function
- Incorporate more interactions into the climate dynamics
- Use optimal control with the pseudospectral method to find carbon emissions scenarios needed to drive global mean surface temperature to a specific point; try several experiments that might be of interest to policymakers

4.0 Models

We try to represent climate dynamics in a state space form where one of our state variables is temperature and the input is related to carbon emissions. Because we are not climate scientists or chemists, we look to published studies to find equations that can help us describe climate interactions. We sometimes reformat these equations in a useful manner via differentiation, subtraction, etc., but we try to use equations from the same published study for consistency.

4.1 Original Model

This work builds off of research done by Justin Ruths and Dr. Jr-Shin Li of the Department of Electrical and Systems Engineering at Washington University in St. Louis. Their original model is a zero-dimensional EBM that evolves the global mean surface temperature of the Earth. Their equations for incoming and outgoing radiation accounted for the effects of atmosphere, and their temperature dynamics account for the effects of increased CO_2 concentrations on temperature. Their model appears below:

$$\dot{T} = \frac{1}{C}\left[\frac{\sigma_s}{4}(a_i + \frac{1}{2}(a_f - a_i)(1 + \tanh(\kappa T))) - A - BT + \frac{\kappa u(t)}{\rho K}\right]$$

$$\dot{K} = \frac{1}{\rho} u(t)$$

where T is the temperature in Celsius, K is the atmospheric CO_2 concentration in ppmv, and u is the anthropogenic carbon emissions in Gt C/year. The Appendix contains a description of all parameters and their values. (Ruths, Li, and Tung, "Global Climate Change").

4.2 New Model

The original model was successful in introducing a systems framework for climate change modeling, but we wanted to find another model that could take more factors into account, such as the effects of aerosols, volcanic eruptions, and carbon sinks on temperature. We originally wanted to start with aerosols, but research quickly demonstrated that this would be difficult; the extent to which aerosols affect climate is still unknown. (U.S. Climate Change Science Program).

After much searching, we found a model that incorporates the effects of carbon uptake by vegetation, soil, and the ocean on carbon concentrations, as well as the effects of water vapor and carbon dioxide concentrations on absorbed radiation. This model, developed by Timothy Lenton and published in the January 2000 issue of *Tellus*, assumes that plants uptake carbon dioxide from the atmosphere via photosynthesis, release it to the atmosphere via plant respiration, and release it to the soil via litterfall:

$$
\begin{aligned}
&\frac{dK_v}{dt} = P(K,T) - R_p(K_v,T) - L(K_v) \\
&P(K,T) = k_p K_{v,ss} k_{mm} \left(\frac{\frac{K}{k_a} - k_c}{k_M + \frac{K}{k_a} - k_c} \right) \frac{(T-273.15)^2(313.15-T)}{5625} \\
&R_p(K_v,T) = k_t K_v k_A e^{-E_a/(RT)} \\
&L(K_v) = k_t K_v
\end{aligned}
$$

where T is temperature in Kelvin, K is the number of moles of carbon in the atmosphere, and K_v is the number of moles of carbon stored in vegetation.

Similarly, the model assumes that soil absorbs carbon from plants via litterfall and releases it to the atmosphere via respiration:

$$
\begin{aligned}
&\frac{dK_s}{dt} = L(K_v) - R_s(K_s,T) \\
&R_s(K_s,T) = k_{sr} + K_v k_B e^{-308.56/(T-227.13)}
\end{aligned}
$$

where K_s is the number of moles of carbon stored in soil.

Many of the constants in these equations have been fitted to pre-industrial or historical data. For a description of all parameters, consult the Appendix.

This publication also uses a different representation of incoming/outgoing radiation to evolve temperature:

$$
\begin{aligned}
&\frac{dT}{dt} = \frac{1}{c}\left[\frac{(1-A)S}{4}\left(1 + 0.75*(\tau(K) + \tau(CH_4) + \tau(H_2O))\right) - \sigma T^4 \right] a_E \\
&where \\
&\tau(CO_2) = 1.73(K)^{0.263} \\
&\tau(CH_4) = 650 \\
&\tau(H_2O) = 0.0126(HP_0 e^{-(L/RT)})^{0.503}
\end{aligned}
$$

where τ(·) is the opacity of a specific gas. The opacity of a gas relates to the amount of light reflected by a gas particle, and thus relates to the amount of light obscured by certain gasses in the air (California Air Resources Board). The equations for incoming radiation allow the opacity of three major greenhouse gases to impact the amount of absorbed radiation, thus allowing for carbon feedback in the temperature dynamics. The equations for water and carbon dioxide opacity are fitted to experimental data using radiative-convective models while the opacity of methane is assumed to be constant at its pre-industrial level. The expression for outgoing radiation assumes that the Earth radiates like a black body and thus is given by the Stephan-Boltzmann law for radiation of black bodies (Lenton).

The model in this paper also accounts for the transfer of carbon dioxide throughout the ocean and into the atmosphere using four boxes: low-latitude surface ocean, high-latitude surface ocean, intermediate depth water, and deep ocean water. We have currently left out this portion of the dynamics because the equations for these dynamics were not easily found in referenced papers.

Thus, our final state space equation is

$$\frac{dT}{dt} = \frac{1}{c}\left[\frac{(1-A)S}{4}\left(1 + 0.75*(\tau(K) + \tau(CH_4) + \tau(H_2O))\right) - \sigma T^4\right]a_E$$
$$\frac{dK}{dt} = u(t) - \frac{dK_v}{dt} - \frac{dK_s}{dt} = u(t) - P(K,T) + R_p(K_v,T) + R_s(K_s,T)$$
$$\frac{dK_v}{dt} = P(K,T) - R_p(K_v,T) - L(K_v)$$
$$\frac{dK_s}{dt} = L(K_v) - R_s(K_s,T)$$

where u(t) are our carbon emissions in moles C/year.

5.0 Experiments: Methods

Using these models, we wanted to demonstrate that we could find emissions scenarios that would drive temperature from one specified point to another. We formulated an optimal control problem and solved it using the pseudospectral method to accomplish this task.

5.1 Optimal Control Problem

We formulated a fixed endpoint optimal control problem to solve for future carbon emissions scenarios that cause desired temperature changes. Given our current temperature/CO_2 concentrations and a final desired temperature, we want to find an emissions scenario that minimizes the value of

$$\int_{t_0}^{t_f} u^T(t)\, u(t) dt$$

subject to our dynamics and endpoint constraints

$$\dot{x}(t) = f(x(t), u(t))$$
$$x(t_0) = x_0$$
$$x(t_f) = x_f$$

The objective function forces us to select an emissions scenario that not only drives the system to the desired point, but also minimizes the total amount of carbon emitted over the time period.

For the original model, our optimal control problem becomes

$$\min_u J(u) = \int_{t_0}^{t_f} u^T(t)\, u(t) dt$$

$$s.t. \;\; \dot{T} = \frac{1}{C}[\frac{\sigma_s}{4}(a_i + \frac{1}{2}(a_f - a_i)(1 + \tanh(\kappa T))) - A - BT + \frac{\kappa u(t)}{\rho K}]$$

$$\dot{K} = \frac{1}{\rho} u(t)$$

$$K(t_0) = K_0$$

$$T(t_0) = T_0$$

$$T(t_f) = T_f$$

$$K(t) \geq 0$$

$$u(t) \geq 0$$

Similarly, the optimal control problem for the new model is

$$\min_u J(u) = \int_{t_0}^{t_f} u^T(t)\, u(t) dt$$

$$s.t. \;\; \dot{T} = \frac{1}{c}\left[\frac{(1-A)S}{4}\left(1 + 0.75 * (\tau(K) + \tau(CH_4) + \tau(H_2O))\right) - \sigma T^4\right] a_E$$

$$\dot{K} = u(t) - \frac{dK_v}{dt} - \frac{dK_s}{dt} = u(t) - P(K,T) + R_p(K_v,T) + R_s(K_s,T)$$

$$\dot{K}_v = P(K,T) - R_p(K_v,T) - L(K_v)$$

$$\dot{K}_s = L(K_v) - R_s(K_s,T)$$

$$K(t_0) = K_0$$

$$K_v(t_0) = K_{v,0}$$

$$K_s(t_0) = K_{s,0}$$

$$T(t_0) = T_0$$

$$T(t_f) = T_f$$

$$K(t) \geq 0$$

$$K_v(t) \geq 0$$

$$K_s(t) \geq 0$$

$$u(t) \geq 0$$

5.2 Pseudospectral Method

The nonlinearities in our dynamics make it difficult to find an optimal input that solves the optimal control problem. Instead of trying to find a general solution, we apply the pseudospectral method to each individual trial run. The pseudospectral method approximates our state variables and control using Lagrange approximation with N+1 nodes. This allows us to transform our constraints from differential to algebraic equations. The method also calls for Gaussian quadrature to get rid of the integral in the objective function. Note: before applying Lagrange approximation or Gaussian quadrature, the pseudospectral method scales the time domain from [0, t_f] to [-1,1]. This implementation of the pseudospectral method follows one introduced in "A Pseudospectral Method for Optimal Control of Quantum Systems" (Ruths, Li, Stefanatos). Consult this publication for a detailed explanation of the implementation.

With this implementation, applying the pseudospectral method to the original problem yields

$$\min_u \quad \left(\frac{t_f}{2}\right)\sum_{k=0}^{N} \bar{u}_k^{\,2} w_k$$

$$s.t. \;\; \frac{t_f}{2C}\left[\frac{\sigma_s}{4}\left(a_i + \frac{1}{2}(a_f - a_i)(1+\tanh(\kappa \bar{T}_j)\right) - A - BT + \frac{\kappa \bar{u}_j}{\rho \bar{K}_j}\right] = \sum_{k=0}^{N} D_{jk}\bar{T}_k$$

$$\frac{t_f}{2\rho}\bar{u}_j = \sum_{k=0}^{N} D_{jk}\bar{K}_k$$

$$\bar{K}_0 = K_0$$

$$\bar{T}_0 = T_0$$

$$\bar{T}_N = T_f$$

$$K_k \geq 0$$

$$u_k \geq 0$$

where $\bar{u}_i$ is the value of u at the i^{th} node, $\bar{K}_i$ is the value of K at the i^{th} node, and D_{jk} is the differentiation matrix.

For the pseudospectral problem for the new model, consult the attached AMPL code.

6.0 Experiments: Trials

We ran several tests to demonstrate that we could find an emissions scenario that would drive temperature from one specified point to another. To illustrate how our method could be useful in developing policy, we ran tests to find emissions scenarios that would cause temperature increases consistent with the projections in the IPCC's Fourth Assessment Report. Using several different emissions scenarios, the IPCC ran simulations to predict temperature trends through the next century ("AR4 SYR Synthesis Report Summary for Policymakers"). We arbitrarily picked 2050 as our target year. Based off of the graphs in the report, it appears that the minimum predicted increase from 2010 to 2050 is approximately 0.65°C while the maximum predicted increase is approximately 1.25°C. The smallest estimate is based off of a scenario where the society follows global environmental sustainability; the largest increase is based off of scenario where the society is globalized and faces rapid economic growth. Our trial runs include:

- IPCC low estimate: An increase of 0.65°C by 2050
- IPCC mid estimate: An increase of 0.95°C by 2050
- IPCC high estimate: An increase of 1.25°C by 2050

In addition to these trials, we also tested other interesting scenarios:

- No increase in temperature: An increase of 0°C by 2050
- Decrease in temperature: A decrease of -0.5°C by 2050
- Original scenario tested by Ruths and Li: An increase of 2°C by 2050

Our experiments were implemented in AMPL using the KNITRO solver, a third-party nonlinear constrained optimization solver. We used all of the default settings, except for the maximum number of iterations, which we increased from 10,000 to 100,000. Our pseudospectral method uses 25 Legendre-Gauss-Lobatto (LGL) nodes over the 40-year period. We use LGL nodes instead of equally-spaced nodes to avoid Runge phenomenon (Ruths, Li, Stefanatos, "A Pseudospectral Method"). More nodes could have been used, but the complexity of the problem increases with more nodes, and this number of nodes seemed to produce relatively accurate results. Once we found optimal emissions scenarios, we put the data into MATLAB to graph and verify our results. To ensure that our optimal emissions scenario drove temperature to the desired point, we used the `ode45` command to evolve our dynamics forward in time. Because our method only finds the control's value at specific points in time and the `ode45` command needed its values at other times, we interpolated between the two nearest nodes using the `interp1` command.

We ran these trials using the original model. We hoped to also run these tests with the new model and compare results. We encountered several difficulties in running the new model and are still in the process of correcting the issues (see Section 7.2).

7.0 Results

This section shows the results for each of our trial runs.

7.1 Results: Original Model

Original Scenario Tested by Ruths and Li: Increase of 2°C by 2050

The solver successfully converged to an optimal solution for this problem. No adjustments of the model's specifications or the solver's options were needed.

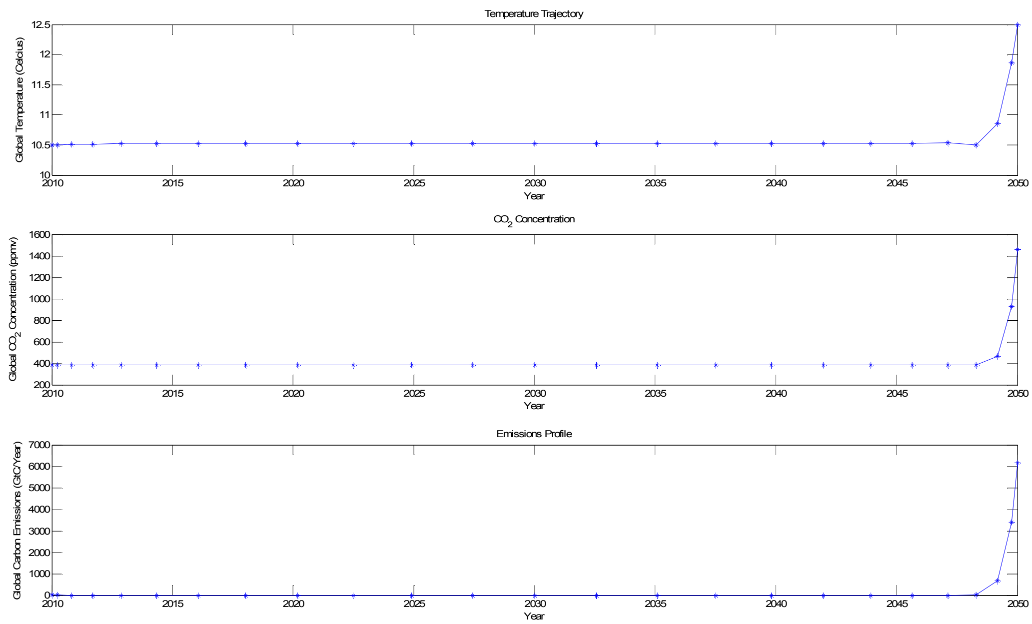

Figure 1: Results for a 2°C increase by 2050 using the original model

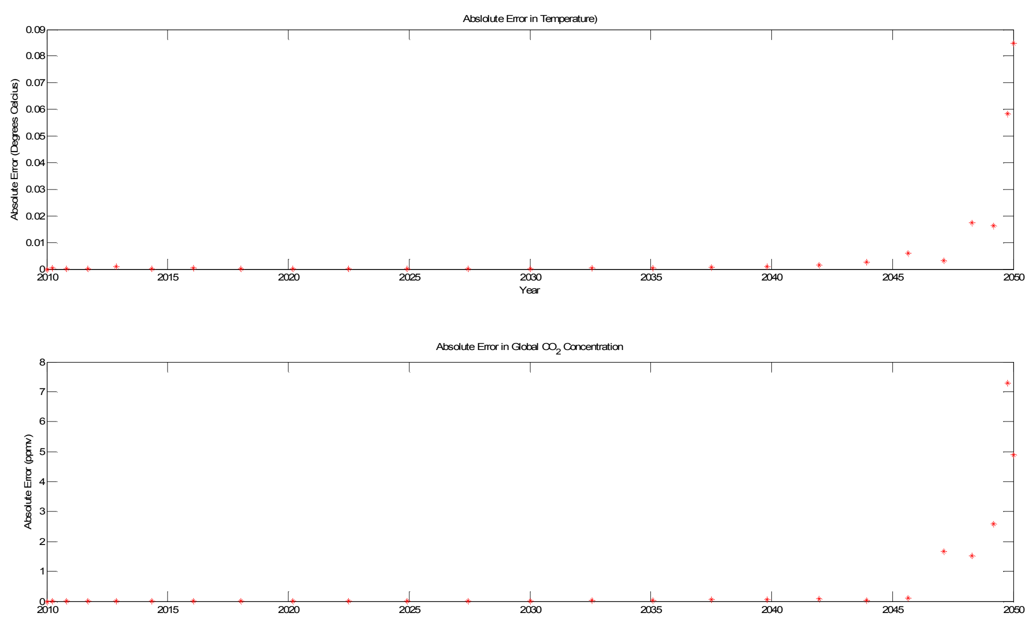

Figure 2: Absolute error at each node for a 2°C by 2050

No increase in temperature by 2050

The solver converged to an optimal solution for this scenario, but the values of the state variables and the behavior of the optimal control were not desirable. We then tried to set a minimum value on temperature in an effort to find a more realistic solution. We set the minimum temperature to 2.5°C below the initial point because we assumed that anything more than this could be just as destructive as significant global warming. The solver could not converge to an optimal solution in the maximum number of iterations, which indicates that the solution is either difficult to find or non-existent. We could not use this emissions scenario to evolve our dynamics using the `ode45` command because of its erratic behavior.

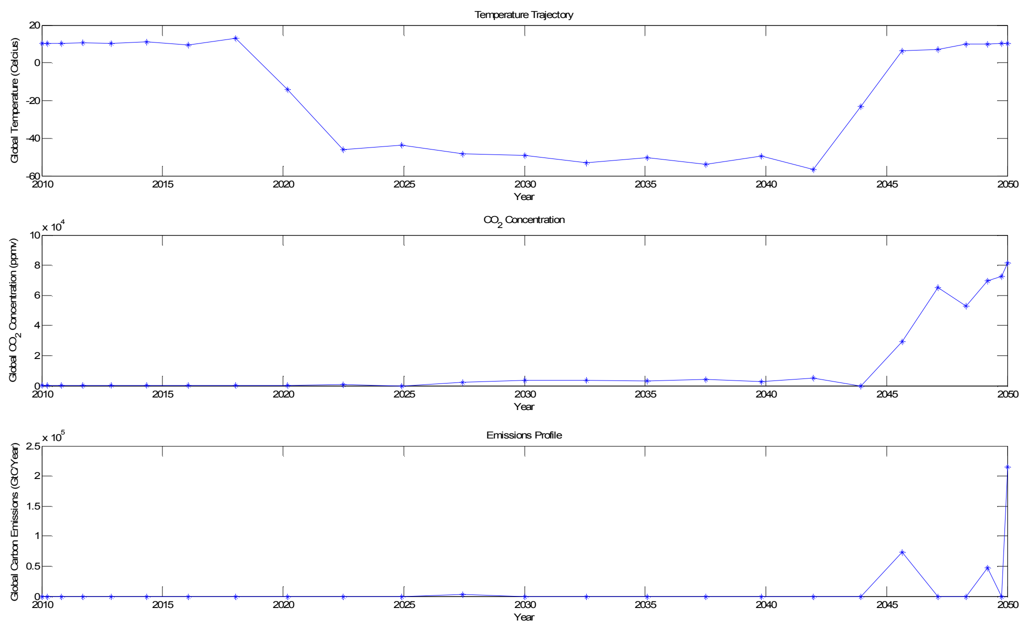

Figure 3: Results for no increase by 2050 using the original model

Decrease in temperature: A decrease of -0.5°C by 2050

The solver could not converge to a solution for this problem in 100,000 iterations.

IPCC low estimate: An increase of 0.65°C by 2050

The solver converged to a solution for this test, but the result was not desirable. We then set a minimum constraint of 8°C on temperature to find a more reasonable result.

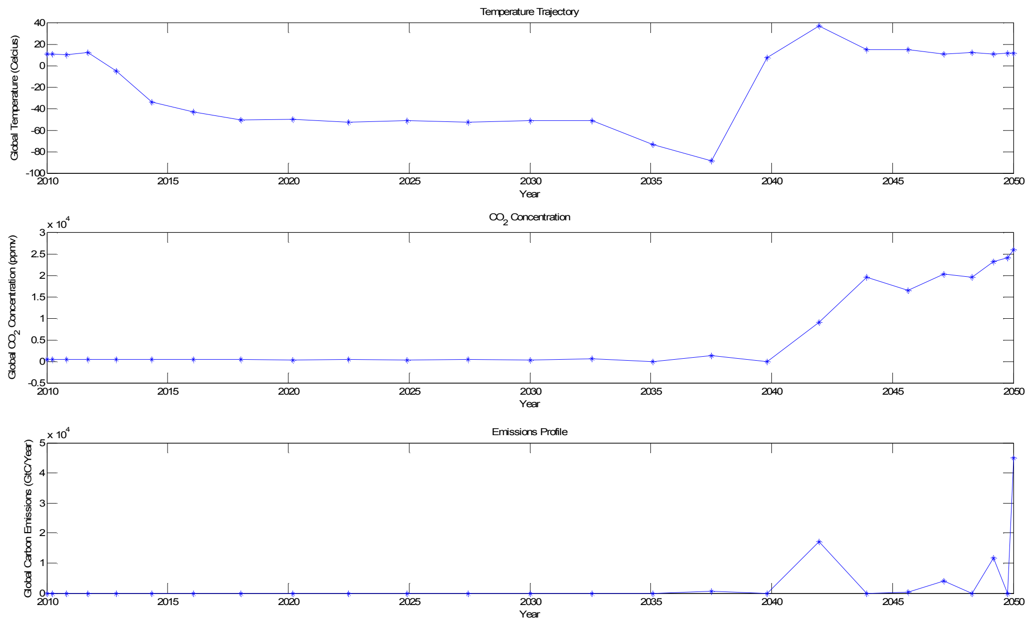

Figure 4: Results for a 0.65°C increase by 2050 using the original model

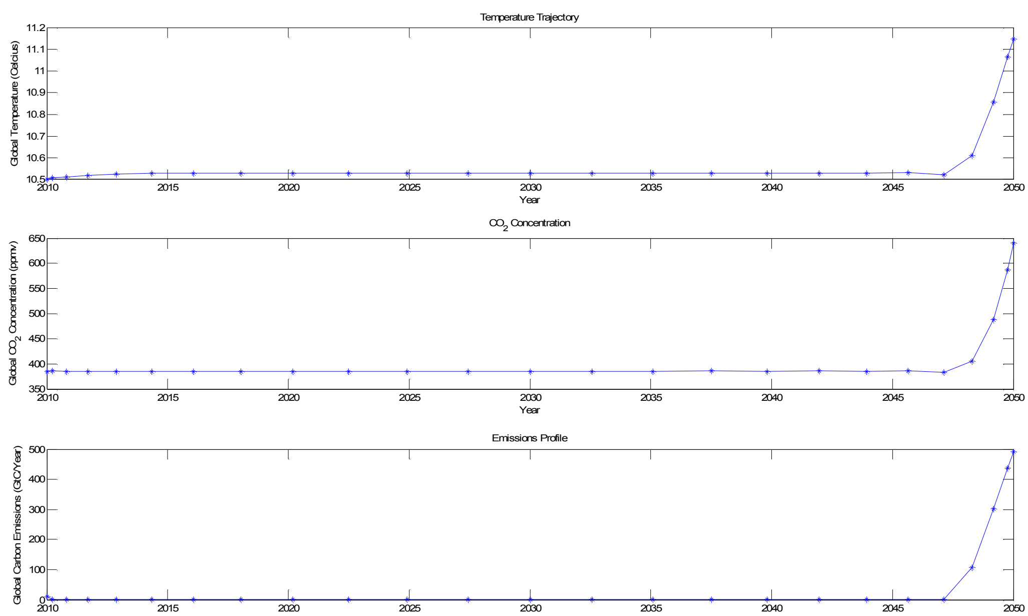

Figure 5: Results for a 0.65°C increase by 2050 using the original model with a constraint on the value of temperature

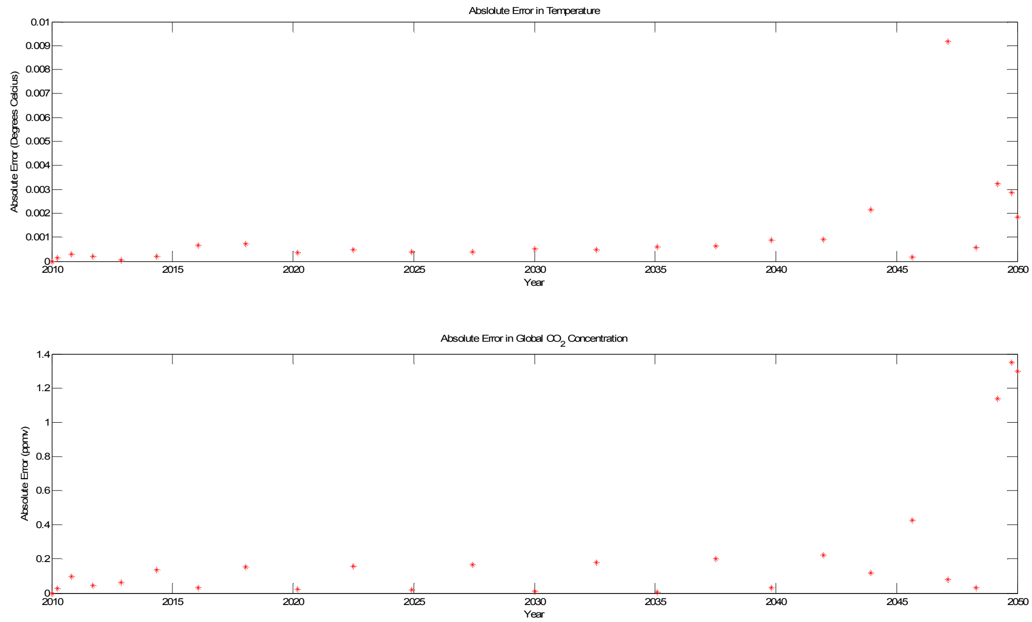

Figure 6: Absolute error at each node for a 0.65°C increase by 2050 using the original model with a constraint on the minimal value of temperature

IPCC Mid Estimate: An increase of 0.95°C by 2050

The solver converged to a solution for this scenario, but the behavior of the state variables was not desirable. We also ran this scenario assuming that temperature could not be less than 8°C to find a more realistic solution.

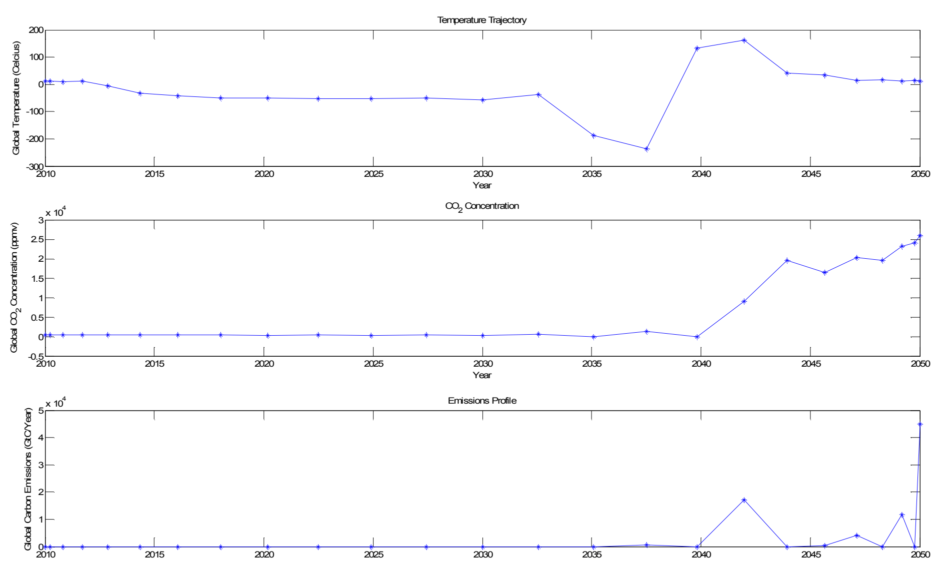

Figure 7: Results for a 0.95°C increase by 2050 using the original model

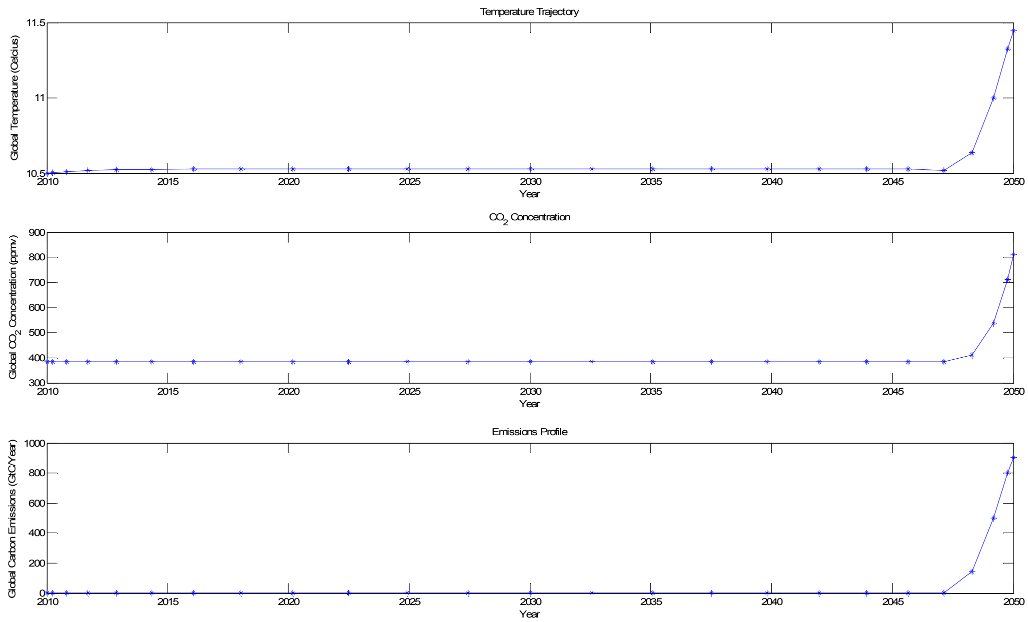

Figure 8: Results for a 0.95°C increase by 2050 using the original model with a constraint on the value of temperature

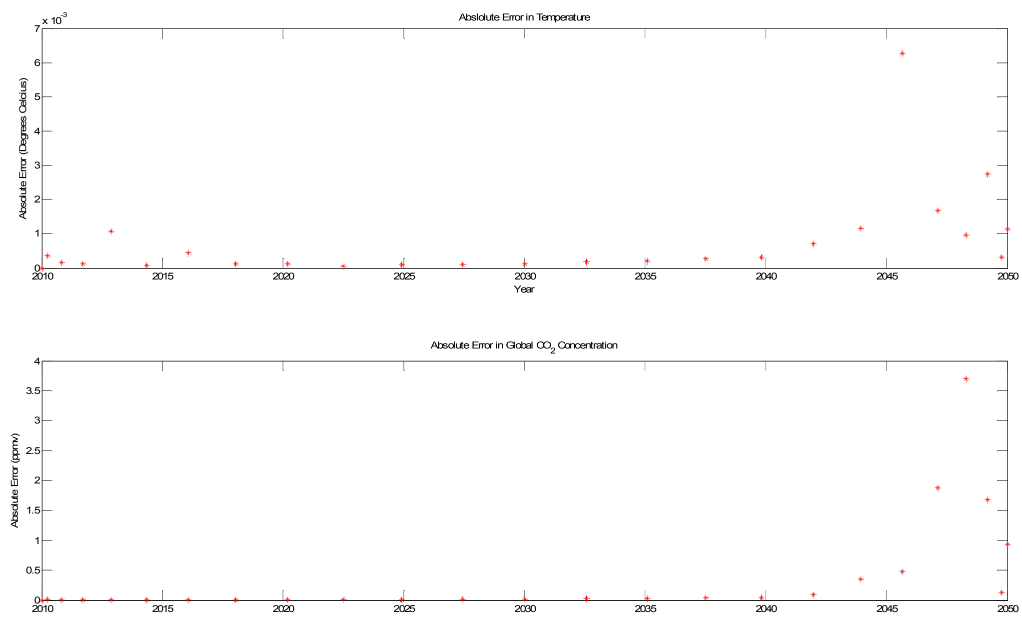

Figure 9: Absolute error at each node (with constrained temperature)

IPCC high estimate: An increase of 1.25°C by 2050

The solver converged to a solution, but the values of state variables were undesirable. We also ran this scenario with a constraint of 8°C on the minimum value of temperature.

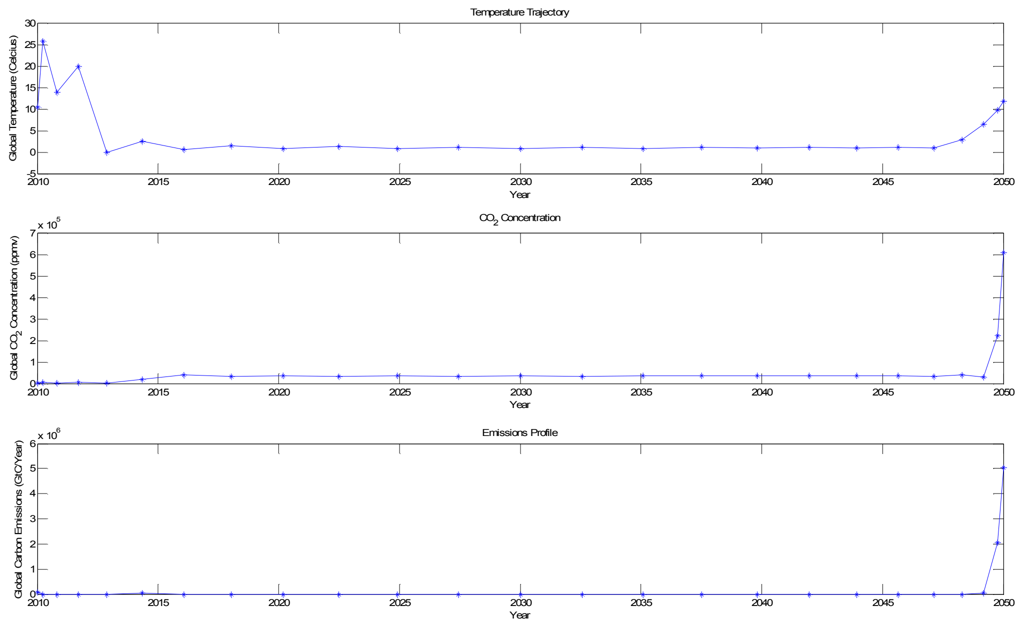

Figure 10: Results for an increase of 1.25°C by 2050 using the original model

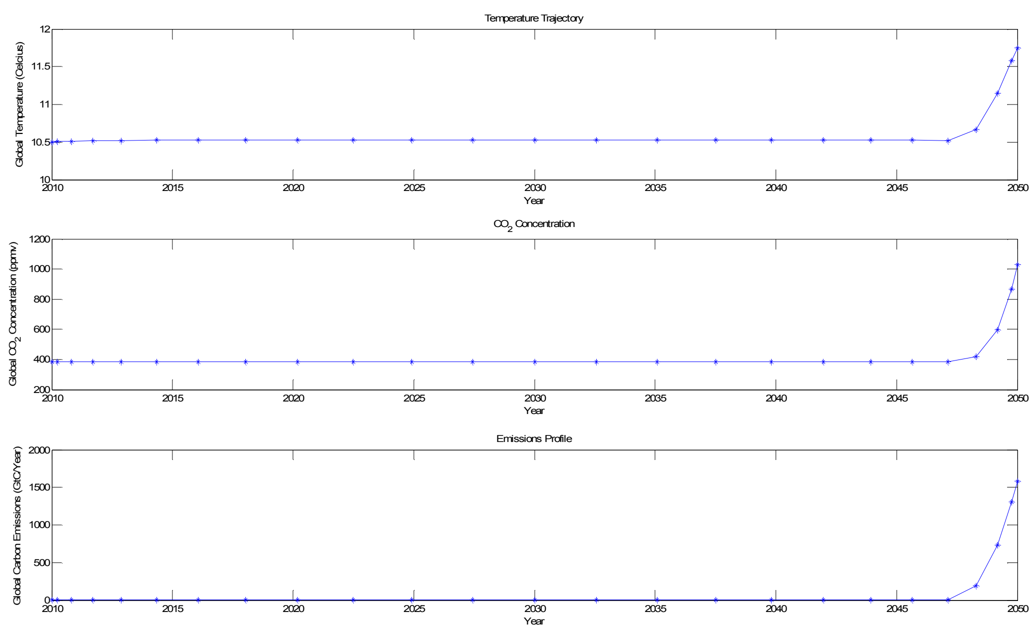

Figure 11: Results for an increase of 1.25°C by 2050 using the original model with constrained temperature

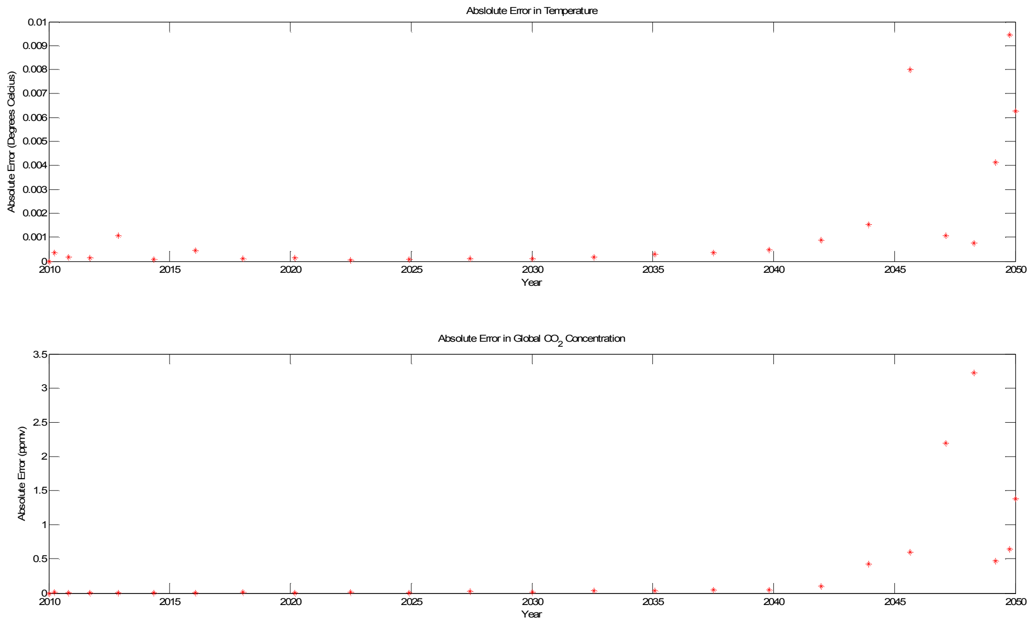

Figure 12: Absolute error at each node (with constrained temperature)

7.2 Results: New Model

When we tried to use the same solver for the new model, we encountered many difficulties and were not able to find solutions. When we ran the model initially, we did not identify a starting point for the solver. The KNITRO solver has the ability to find its own starting points, but it was not able to find a starting point at which all equations could be evaluated. To alleviate this problem, we evolved the dynamics from the initial conditions assuming an input of zero using ode45 in MATLAB. We then interpolated to find the value of the state variables at each of the N+1 nodes and used these values as our starting points for the optimization problem. While this got us past initial hurdles, the solver continued to have problems. After a few hundred iterations, the solution got stuck in an infeasible region; the optimal value could not be improved by more than our minimum tolerance, and the solution at this point was infeasible. We believe that the magnitude of our state variables could be the source of this problem; the values are based off of moles of molecules and thus are large in size. The next step in trying to make it work would be to scale our dynamics to get smaller, more manageable numbers. We are currently in the process of finding the right scaling factor.

8.0 Discussion

Our experiments indicate that our method is capable of finding emissions scenarios that would theoretically drive temperature to a desired point. Some important features to note include:

- The solver converged to a relatively well-behaved solution for the 0.65°C, 0.95°C, 1.25°C, and 2°C increase scenarios. The optimal emissions for these test runs all had similar shapes, most likely because this shape is consistent with trying to minimize the area under the emissions curve.
- The absolute error at each node for both temperature and CO_2 concentration are reasonable in size. It is important to remember that these errors could be due to approximations in the pseudospectral method as well as numerical errors introduced by the interpolation and differential equation solver methods we used to evolve the model forward in time.
- The solver did not converge to reasonable solutions for the 0°C and -0.5°C scenarios in the allotted number of iterations. This could indicate that it would be difficult to find an optimal solution for these tests or that one does not exist.
- At first glance, the optimal emission scenarios we were able to find seemed to lend themselves well to carbon emissions policies; they are relatively smooth, increasing functions. Further investigation revealed that restricting emissions to these levels would not be feasible to implement because our current global anthropogenic carbon emissions are over 30 GtC/year. We could potentially set constraints on the minimal and maximal values of our control to find a more realistic scenario.

9.0 Conclusions

We were successful in finding a new description of climate dynamics that accounted for more factors than our original model. While we are still in the process of formatting this model so that it is compatible with the KNITRO solver, we were able to use the original model and demonstrate that optimal control could be a useful tool in developing global warming policy. The pseudospectral method allowed us to solve the optimal control problem with a reasonable amount of accuracy.

On a less technical note, I can now understand why few systems scientists are in the climate modeling field. Systems scientists must rely on climate scientists or publications to find climate models to which they can apply systems theory. Unfortunately, trying to find a model that could be formatted in state-space form proved difficult, and finding a publication that included all parameters and equations was even more challenging. After meeting with a professor in the Earth and Planetary Sciences department, it became apparent that this occurs partly because climate scientists have limited knowledge about what insight systems theory can provide. If systems science is to become an important tool for climate modeling, there needs to be more collaboration and communication between systems scientists and climate scientists.

I will be continuing this project in the future, and two undergraduate students will be joining our team over the summer. The next step in our research will be to get our new model to run with the KNITRO solver and also incorporate ocean uptake of carbon dioxide. In addition, we may be interested in using a radiative forcing model to describe climate dynamics. The IPCC lists predictions on the amount of radiative forcing characteristic to various substances, so using this model would allow us to incorporate the impacts of more substances on climate. Also, because the IPCC lists a range of radiative forcing predictions, it may be interesting to derive the sensitivity equations for these parameters and investigate the sensitivity of solutions to changes in these parameters.

10.0 Acknowledgements

I would like to thank Justin Ruths and Dr. Jr-Shin Li for taking the time to advise me throughout this research project and helping me overcome difficulties in the research process.

11.0 Appendix

The following tables summarize all parameters used in our models.

A	Fitted Constant	218 W/m^2
B	Fitted Constant	1.9 $W/°C\text{-}m^2$
C	Specific Heat Capacity	3.52 $W\text{-}yrs/°C\text{-}m^2$
κ	Fitted Constant	6.5
ρ	Density of Carbon in Atmosphere	2.13 GtC/ppmv
Q	Solar Input Constant	340 W/m^2
a_i	Ice Coalbedo	0.35
a_f	No-ice Coalbedo	0.7
γ	Fitted Constant	0.9

Table 1: Parameters for Original Model. Adapted from Lenton's "Feedback Effects on Global Warming."

k_p	Photosynthesis Rate Constant	0.184/yr
k_r	Plant Respiration Rate Constant	0.092/yr
k_t	Turnover Rate Constant	0.092/yr
k_{sr}	Soil Respiration Rate Constant	0.0337/yr
k_c	Compensation Point	29 ppmv
K_M	Half-Saturation Point	120 ppmv
E_a	Plant Respiration Activation Energy	54830 J/mol
k_{MM}	Photosynthesis Normalizing Constant	1.478
k_A	Plant Respiration Normalizing Constant	8.70E+09
k_B	Soil Respiration Rate Constant	157.072
S	Solar Flux	1368 W/m^2
A	Surface Albedo	0.225
P_0	Water Vapor Saturation Point	1.4E+11 Pa
L	Latent Heat Per Mol Water	43655 J/mol
R	Molar Gas Constant	8.315 J/mol-K
H	Relative Humidity	0.6177
c	Specific Heat Capacity	4.69E+23 J/K
aE	Surface Area of the Earth	5.101E+14 m^2

Table 2: Parameters for New Model. Adapted from Lenton's "Feedback Effects on Global Warming."

Please contact Jess Stgile at jls2@wustl.edu for AMPL and MATLAB code.

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