Feedback Control of Climate Dynamics

Jessica Stigile

Advisors: Justin Ruths and Dr. Jr-Shin Li

Department of Electrical and Systems Engineering, Washington University in St. Louis

Abstract

Climate change has become a topic of great importance in recent years. Interest in climate change has even reached the government level as policymakers are looking for ways to mitigate the effects of global warming. We are interested in studying complex climate models with carbon-cycle feedbacks using control theoretic techniques. Formulation of an optimal control problem and solution via the pseudospectral method give insight into emission scenarios needed to drive global mean surface temperature to a specific value by a specific date.

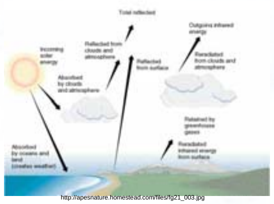
Background

Climate Models

- Mathematically describe the evolution of a climate characteristic over time, usually temperature
- Aid decision-makers in developing plans to mitigate global warming

Energy Balance Models (EBMs)

- Describe the evolution of climate based off of the incoming and outgoing radiation



http://apesnature.homestead.com/files/fg21_003.jpg

- In the presence of atmosphere, incoming radiation is

$$R_{in} = \frac{\sigma_s}{4}[a_i + \frac{1}{2}(a_f - a_i)(1+\tanh(.9T))]$$

where σ_s is the solar constant, and a_i and a_f are the ice and no-ice coalbedos

- Outgoing radiation is

$$R_{out} = A + BT;$$

where A=218 W/m^2; and B=1.9 W/°C-m^2

Model

We use a zero-dimensional Energy Balance Model (EBM) to describe the evolution of global mean temperature based off of the difference in incoming and outgoing radiation:

$$C\frac{dT}{dt} = R_{in} - R_{out}$$

where C=3.52 W-yr /°C-m.

Our model also includes the contribution of increased carbon concentration to increased temperature:

$$\left.\frac{dT}{dt}\right|_{CO_2} = 6.5\frac{\dot{K}}{K}$$

where K is the atmospheric CO_2 concentration in ppm.

State space model:

$$\dot{T} = \frac{1}{C}[R_{in} - R_{out} + 6.5C\frac{\dot{K}}{K}]$$

$$\dot{K} = \frac{1}{\rho}u(t)$$

where u(t) are the annual carbon emissions (GtC/yr) and ρ=2.13 GtC/ppm.

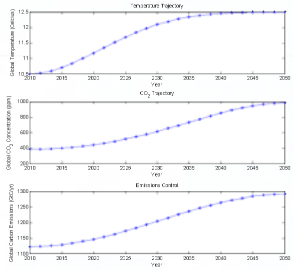
Experiments and Results

By formulating an optimal control problem, we try to reconstruct an emissions profile that would drive the global mean temperature to a specified value:

$$\min_u J(u) = \int_{t_0}^{t_f} u^T(t)\, u(t) dt$$
$$s.t. \quad \dot{x}(t) = f(t, x(t), u(t))$$
$$x(t_0) = x_0$$
$$x(t_f) = x_f$$

We solve this problem using the pseudospectral method.

Example: Starting at our current global mean surface temperature of 10.5°C and CO_2 concentration of 385 ppm, can we hit a target global mean temperature of 12.5°C by 2050?



Recent Work

- Currently incorporating the effects of soil, vegetation, and ocean uptake of atmospheric carbon
- Also interested in incorporating the effects of anthropogenic aerosols into the model

References

J.A. Ruths and J. Li, "Global climate change: control theory methods for a coupled climate model with carbon-cycle feedbacks."

M.I. Budyko, "The effect of solar radiation variations on the climate of the earth." *Tellus*, vol. 21, pp. 611-9, 1969.