Developing an Optimization Tool for Selecting Home Investment Strategies

For ESE 499 – Senior Design Project
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Company B works with homeowners to help them reduce their utility bills. They do so through an energy audit, which involves an expert visiting the home and identifying opportunities for the owner to save energy. Currently, the service does not include any energy savings estimates that are specific to the home's existing condition. To improve the energy audit, Company B would like to have a tool that they can use to assess which energy-saving opportunities should be top priority for the homeowner, as well as the detailed financial implications of those investments. To do so, an optimization tool has been developed, using Excel Solver as a platform, to meet the homeowner’s goals (ROI, Savings, Payback Period), while staying within constraints (budget, possible of system combinations, etc). The optimization uses client-specific input, combined with researched assumptions, to provide detailed estimates of savings, return on investment, and payback periods. This information is used to prioritize investment strategies and results in a list of recommendations for the client.
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Background

Company B is a St Louis start-up company that works with homeowners to reduce their utility bills. The company provides two main services: energy audits for existing homes, and consulting for new and major rehab projects. For this project, we will focus on the audit offering.

An audit is a service for homeowners who are just beginning to consider energy efficiency and how it affects the utility bills they pay every month. It consists of an energy expert coming to the home, discussing the homeowner’s priorities and any limitations they may have (such as budget), and then a home inspection to identify problem areas. Throughout the visit, the energy expert is making suggestions of possible improvements, but these recommendations are not prioritized for the homeowner. Currently, the homeowner does not end up with any personalized reports, simply the conversation and some basic information to get them started.

This presents a problem for Company B because some homeowners regret not having a personalized, prioritized list of improvements to make in their home, resulting in poor customer relations. To address this problem, Company B would like to develop a quick and easy tool to help the energy expert create such a report. The challenge is time, since an expert’s time is expensive and the current price of the audit does not cover expert time beyond the current audit setup.

Thus, this senior design project tackled the challenge of developing a tool that incorporates the basic elements of an audit and uses optimization to suggest a basic priority list for a client, subject to the client’s preferences as well as budget and time constraints. It should be easy to use, run quickly, and produce a report that can be e-mailed or printed for the client.
Tool Requirements

There are several key requirements that must be met in order for the tool that is developed to be successful. Here, I define success to be conditioned on its actual implementation into the audit process, and its continued use into the future. Key requirements for the tool include:

- **Technical** – The tool must run on a platform that is easily transferred between computers and runs easily on a laptop so that it can be carried to a job site.

- **User interface** – Since energy experts do not necessarily have the computer skills required to operate a complex tool, the user interface must be simple to understand, easy to operate, and quick to learn.

- **Flexibility** – To be used on every audit, the tool needs to be able to handle a wide variety of potential situations, adjust calculations and estimates of operating costs, and project future savings.

- **Data balance** – In order to be useful, the tool must require fairly detailed information about the home being analyzed, to ensure that the results are relevant to the client. However, the quantity of data required must not exceed that which can be gathered in approximately 15 minutes, to ensure that using the tool does not raise Company B’s cost of conducting an audit.

Development Process

There are several steps necessary for completion of a successful tool development process. They included scope definition, identification of the alternatives to be included as investment options, choice of objective function parameters, data and assumption research, and model design and development.

1. **Scope Definition**

The first step was to define the scope of the project, to ensure the resulting tool met the requirements discussed above. The question was which aspects of an audit would be included in the optimization. This was difficult because there are many
components that contribute to how much energy a home consumes, some of which need to be included (such as HVAC systems), some of which may be superfluous (like a dishwasher), and some that are simply too difficult to incorporate into a standardized model (such as windows). Eventually, I settled on four categories: HVAC systems, domestic hot water heaters, major appliances, and add-on features.

2. Alternative Identification
The next step was to identify the existing alternatives for each category being considered, and then narrow down the many options to those appropriate for consideration. Factors that played into the decision of whether or not to include an alternative were prevalence in existing St Louis homes, investment cost, feasibility of implementation in an existing home, and public availability of performance data. (see Exhibit 1 for a list of all systems included.)

3. Objective Function Parameters
To ensure the accuracy and usefulness of the model, the objective function was carefully thought out. There were several parameters considered as the objective for the entire optimization: maximization of total savings, maximization of return on investment, and minimization of payback period. Any of these three could be legitimate goals for the client, but none of them offered an alternative that would satisfy every client.

4. Research
Given the many alternatives that were identified as relevant for inclusion in the tool, there was a lot of research into the assumptions were used for calculating the objective function parameters. Some example data included average installed efficiencies for HVAC systems, system decay rates, replacement costs, high-end system efficiencies, etc. Unfortunately, most of the publicly available data is scattered in different sources and very little was originally in a form that fit neatly into the model as designed. These issues, and the workarounds that I used, will be discussed in depth in a later section.
5. **Model design and development**

The model itself was clearly going to be an optimization from the start, since I was developing the tool to make recommendations of which alternatives (decision variables) should be selected to meet a specific goal (objective function). With that decision, I faced several possible software programs that could be used to run the optimization. The conclusion was quickly reached that Excel Premium Solver was the optimal package, since it uses a genetic algorithm but is still user friendly and easily run on a laptop on-site at an audit. The actual model design specifics will be discussed in further detail in a later section.

**Assumption Research**

**Sources**

To gather the necessary data, I had to do quite a bit of research. Sources included online databases, such as those on the Energy Star website, rules of thumb from the building industry, phone calls to St Louis product installation companies, and multiple visits to Home Depot. I chose to use their prices and restrict myself to the models available there because it is the most likely location a homeowner would go to purchase what they need to follow Company B’s recommendations. To project energy savings, I also simulated different HVAC configurations with the software package Energy-10 (a program developed by the DOE, used to project energy usage of a given building). See Exhibit 2 for details on using Energy-10.

**Rectifying discrepancies**

There were several instances were data found in different sources suggested very different results. The best example of this was when I researched “annual savings” from using a setback thermostat. According to Home Energy Saver, a setback thermostat should save approximately 300 kWh/yr. According to GreenAndSave.com, it should save 2,045 kWh/yr. Finally, my own simulation using Energy-10 suggested savings of 5,012 kWh/yr. In this case, I chose the middle
estimate, and followed the same strategy of moderation in most cases where discrepancies occurred.

**Data manipulation**

Because I was gathering data from many sources, finding information in various forms, and occasionally having to combine data from several sources to get usable information, there was a fair amount of data manipulation done to define assumptions.

“**Expected savings**”

This was a term frequently used on packaging material, in product advertising, and on general home improvement online resources. This posed a problem, since these claims were based on assumptions such as electricity rate, home size and climate zone, frequency of use, and how long a product would last. Of course, these assumptions were not usually clearly stated and were often skewed because the results were being used for marketing. In most cases, I was able to read the “fine print” and back out the necessary information, usually kWh used per year.

*For example, on a package of CFL light bulbs at Home Depot, the package suggested I could “save $184” by spending the $5.85 to purchase them. In fact, that was an expected total savings for 4 bulbs, at a $.10 per kWh rate, over 9 years, without factoring in the time value of money; thus, the annual savings per bulb was actually 51 kWh per year.*

**Minimum requirements.**

One of the major sources of information on installation costs and relative performance of appliances was Home Depot. In choosing among the appliances that they had on display, the first requirement was that the appliance be Energy Star certified, since the incremental cost is less than the present value of the energy savings over the life of the appliance. The real challenge was identifying which of the many options represented the best purchase, and how much of the price...
differences between models was due to additional features and how much was due to the increased efficiency.

For example, in choosing between refrigerators, there were many options ranging in price from $1,199 to $1,799, with the most expensive refrigerator also using the fewest kWh per year. The question was whether it was actually worth investing an extra $600 for a marginal amount of savings, and how much of the extra cost was because of the efficiency gain and how much because it was a bottom freezer rather than a side-by-side design. In this case, I chose to include both design styles in the optimization, to allow for real option comparison, rather than making an arbitrary decision.

Misfit information.

Another challenge was adjusting information that I did gather to fit into the way I had designed the optimization to work. In fact, the model was redesigned several times to adjust how the calculations were done, on account of such misfit information. Often, I encountered spotty data because standards or testing results are only updated every few years for certain products. Additionally, much of the online information on HVAC system efficiency was not labeled by year, or by relative performance standards, simply as “this system is 85% efficiency”. Particularly once I had implemented the model to account for installation year, this type of data was misfit for the model I had built.

My work to estimate clothes washer energy use is a prime example of having to use misfit data. There is very little data on how much energy a clothes washer uses, especially as a function of when it was installed. In this case, the only misfit information I had was on Energy Star requirements (MEF) at several points in time when the requirements had been updated. To use this information, I first fit an exponential regression (R2 = .973) to the requirement points that I had to estimate MEF over time; exponential was used because it would reflect the increasing speed of technological evolution that leads to efficiency and also so that it would never be a negative value. Using the fact that Energy Star requirements are designed to limit the certification to only the top 15% of appliances, and assuming a uniform distribution of
appliances along the efficiency scale, I was able to calculate the mean MEF for each year. Finally, using the definition of MEF and a user input of how many loads they run, I calculated the yearly kWh used by a washer, given any age.

Model Design Development

To understand the way the model design developed, there is first a discussion of the basics: decision variables, constraints that were eventually included, and the platform used for development, in this case Excel Solver. Following the initial design, there were several phases of design and redesign. Instead of describing each one individually (which would be rather boring and tedious), I have chosen to discuss the design flaws that each sought to address and conclude with a description of the final design.

Decision Variables

In this scenario, I chose to assign each alternative (electric furnace, central air conditioning, standard electric stove) a decision variable representing the future condition of the home. I could then compare the “future” with the current configuration and assess which investments should be made. Given that my four categories of alternatives (HVAC, domestic hot water heater, major appliances, and add-on features) all have different characteristics, the type of decision variable was not going to be the same for each one. The table below shows the decision and rationale.

<table>
<thead>
<tr>
<th>Category</th>
<th>Variable Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heating (H)</td>
<td>Binary – installed (1), not installed (0)</td>
</tr>
<tr>
<td>Cooling (C)</td>
<td>Binary – installed (1), not installed (0)</td>
</tr>
<tr>
<td>Hot water (W)</td>
<td>Binary – installed (1), not installed (0)</td>
</tr>
<tr>
<td>Refrigerator (R)</td>
<td>Binary – installed (1), not installed (0)</td>
</tr>
<tr>
<td>Clothes Washer (CW)</td>
<td>Binary – installed (1), not installed (0)</td>
</tr>
<tr>
<td>Insulation (N)</td>
<td>Integer – number of additional inches installed</td>
</tr>
<tr>
<td>Add-ons (A)</td>
<td>Integer – quantity purchased or in use</td>
</tr>
</tbody>
</table>
Constraints

There were four types of constraints that were eventually included. They included single-system constraints, physical constraints, convenience constraints, and client-imposed constraints.

Single-system

These constraints were needed to ensure that only one alternative for a given system could be chosen. For example, there can only be one heating system in a house, so the sum of the heating system decision variables was restricted to equaling one.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>total number heating systems</td>
<td>1 = 1</td>
</tr>
<tr>
<td>total number cooling systems</td>
<td>1 = 1</td>
</tr>
<tr>
<td>total number hot water</td>
<td>1 = 1</td>
</tr>
<tr>
<td>total number refrigerators</td>
<td>1 = 1</td>
</tr>
<tr>
<td>total number stoves</td>
<td>1 = 1</td>
</tr>
<tr>
<td>total number clothes washers</td>
<td>1 = 1</td>
</tr>
</tbody>
</table>

Physical

These constraints were needed to ensure there were not any inconsistencies within the model. For example, limiting the decision variable for ceiling fans to not exceed the number of appropriate rooms, or not allowing the tool to suggest buying a hot water heater blanket if the house already has one, since you can't use multiple!

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>can't have desuperheater wo/ geothermal HVAC</td>
<td>1 &lt;= 1</td>
</tr>
<tr>
<td>can't add more fans than have space</td>
<td>4 &lt;= 4</td>
</tr>
<tr>
<td>can't replace more incandescents than exist</td>
<td>10 &lt;= 10</td>
</tr>
<tr>
<td>can't install set-back if already one there</td>
<td>1 &lt;= 1</td>
</tr>
<tr>
<td>can't use power strips you don't own</td>
<td>2 &lt;= 2</td>
</tr>
<tr>
<td>can't caulk if home already tight</td>
<td>0 &lt;= 0</td>
</tr>
<tr>
<td>can't buy blanket if already own one</td>
<td>0 &lt;= 0</td>
</tr>
<tr>
<td>can't add ducting insulation if already there</td>
<td>1 &lt;= 1</td>
</tr>
</tbody>
</table>

Convenience

These constraints were designed to prevent investment recommendations that go against natural preferences, such as not allowing multiple types of attic insulation, since this would cause a mess and aggravation.
Client-imposed

These constraints consisted of limits on the client’s budget and tolerance of longer payback periods (a reflection of their risk tolerance).

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>can’t replace central air with window units</td>
<td>0</td>
<td>&lt;= 0</td>
</tr>
<tr>
<td>can’t add central air to buildings wo/ ducting</td>
<td>1</td>
<td>&lt;= 2</td>
</tr>
<tr>
<td>can’t add central heat to buildings wo/ ducting</td>
<td>1</td>
<td>&lt;= 2</td>
</tr>
<tr>
<td>only one type of insulation</td>
<td>1</td>
<td>&lt;= 1</td>
</tr>
</tbody>
</table>

Excel Solver

Using Excel Solver presented some unique challenges on several fronts. First, there were some basic spreadsheet design constraints; since Solver is a fairly basic piece of software, it requires that decision variables, constraint parameters, and objective functions all be contained in the same sheet. Given the large number of calculations done to estimate total savings, this meant extensive pre-planning for how they would be organized throughout the workbook. Second, Solver does not always do very well at locating global maxima, instead getting “stuck” at local maxima, because it uses a hill-climbing algorithm. For this reason, I chose to use the software extension package, Premium Solver. Third, because of the algorithms that Solver uses, the use of complex functions such as if/else and min/max tends to cause Solver trouble in finding global optima; thus, after writing the entire tool the first time, I then revisited all of the calculations to remove if/then and min/max statements, primarily by substituting binary variables that could be used to the same effect.

Initial Design

The initial design was for the optimization to be set up as described below. There were several major flaws in the initial calculations, including an inability to factor in the extra savings from add-on features, system age, and initial installed efficiency. Decision variables:

\[ x_{ij} = \text{decision variable for category } i, \text{ alternative } j \]
Objective function:

Maximize total savings ($)

Subject to:

\[ \sum x_{Hj} = 1 \quad \sum x_{Rj} = 1 \]
\[ \sum x_{Cj} = 1 \quad \sum x_{CWj} = 1 \]
\[ \sum x_{wj} = 1 \quad \sum x_{nj} = 1 \]

Equations:

Total Savings = \[ \sum_{i=H}^{A} S_i \]
\[ S_i = (oc_{existing} - \sum_{j\in i} x_{ij} \cdot oc_{ij}) \cdot l_i \cdot \prod p_{ij} \cdot x_{ij} \]

Where

\( l_{ij} \) = expected lifespan of alternative \( ij \)
\( p_{ij} \) = purchase price of new system \( ij \)
\( oc_{ij} \) = operating cost of system \( ij \)

Adjustments

To address the many issues with the initial design, misfit data, and improve accuracy of the model, several key adjustments were made. Each is described below, in approximate implementation order.

Objective function improvement.

In the initial model, the objective function was simply maximization of total savings. However, there were several parameters that could be considered: maximization of total savings, maximization of return on investment, and minimization of payback period. Any of these three could be legitimate goals for the client, but none of them offered an alternative that would satisfy every client. Thus, I chose to allow client input in the form of a ranking of the three parameters. I then combined these weights with their respective parameters to obtain an objective function that was tailored to client priorities.
The objective function has three key features that allow for this combination of priorities. First, I could not simply multiply the rankings by their parameters, since this would result in the opposite of the client’s priorities, since the lowest ranked parameter would be weighted the highest. Instead, I used multiplied the parameters by the inverse of their rank to address this issue. Second, in order to include parameters that need to be both maximized and minimized simultaneously, the parameters could not simply be multiplied by their weight and then added together. Instead, I chose to maximize the entire function while subtracting the payback period, with an end result of minimizing the payback period as desired. Third, because of the relatively wide scale of the parameters, there was an unintended weighting of total savings (in thousands of dollars) over payback (in single-digit years) and ROI (in percentage points). To remedy this, I chose to scale all of the parameters to single-digit values (dividing savings by 10,000 and multiplying ROI by 10).

**Savings per year calculation.**
To incorporate the insulation and add-on features correctly, the total savings calculation for these two categories had to be adjusted. Whereas for the Heating, Cooling, and Hot Water categories, savings were calculated on a comparison to the current cost baseline (current costs – future costs), there is no comparable baseline for insulation or add-ons. Thus, their contribution to Total Savings was calculated based on yearly savings that could be achieved – for example, by adding a ceiling fan and adjusting the thermostat accordingly, one can save $26 per year, over 13 years, for a total savings contribution of $338.

**Additional constraints.**
As the optimization became more complicated, additional constraints (beyond those in the initial design) were necessary, particularly when considering add-on features. These resulted in the physical, convenience, and client-imposed constraints discussed earlier.
Developing an Optimization Tool for Selecting Home Investment Strategies

Efficiency as a function of age.
Based on my research, there have been significant improvements in heating, cooling, and water heating system efficiency over the past twenty years, resulting in significant differences in energy usage between systems that were installed in the 1980’s and those installed today. Thus, I concluded that my assumptions of operating costs needed to factor in the system's age. This was done by using the vlookup() Excel function to auto-select the system’s efficiency out of one table, and then use that efficiency to estimate operating costs. These operating costs were then used to estimate total savings. Note that there is the option in the model for clients to indicate the actual efficiency of their system; if they do so, that efficiency is used instead.

System degradation.
As I developed the model, my research showed that heating, cooling, and hot water heating system efficiency drastically decreased over time. Thus, I incorporated a degradation factor into my calculations. The estimated system efficiency was found as a function of age (see previous adjustment), then degraded at a 1.2% rate per year. For example:

Natural gas boiler, 13 years old.
Estimated efficiency = 80% AFUE
Adjusted efficiency = (80%) * (1 - .012)^13 = 68.4% AFUE

Complex function elimination.
As I began testing the optimization, I found that I was sometimes seeing suboptimal results. Eventually, I discovered that the problem lay with the fact that Solver uses a hill-climbing algorithm. It works by testing nearby solutions; if one of them results in a better objective function, it is chosen as the new solution. When complex functions, such as minimum(), maximum(), or if/else() are used in the optimization, this results in the hill-climbing algorithm sometimes settling at local maxima, rather than the problem optimum. Thus, I had to rebuild the component calculations to avoid using these problematic functions.
Net Present Value (NPV).

Two factors that play into a homeowner's decision making process, human nature to wait to spend money (discounting future value) and rising fuel costs (resulting in rising operating costs), I chose to use Net Present Value (NPV) equations to calculate total savings. Standard NPV calculations discount the future value of money relative to an interest rate i, over t years. The equation is:

\[
NPV = A \left( \frac{1 - \frac{1}{(1 + i)^t}}{i} \right)
\]

In this case, rising fuel costs also need to factor in, so the adjusted NPV is calculated by including a factor g to represent the growth factor:

\[
NPV = \frac{A}{i - g} \left( 1 - \left( \frac{1 + g}{1 + i} \right)^t \right)
\]

To apply these equations to this problem, I also had to consider the fact that by replacing an existing system now, there would be one level of savings before the system would have been replaced anyway, and a different level of savings after the system would have been replaced. (This is common sense – it is more worthwhile to replace a really old system that is going to die anyway.) To reflect this, several NPV calculations were combined to reflect this reality. The actual savings are depicted below (left) next to the calculated savings (right). The NPV is calculated by taking an NPV of the blue savings for the full 20 years, then adjusted by subtracting out an equal NPV over the first 7 years.
The NPV equation becomes:

\[
NPV = \frac{Red}{i-g} \cdot \left[ 1 - \frac{(1+g)^{t_{Red}}}{1+i} \right] + \frac{Blue}{i-g} \cdot \left[ 1 - \frac{(1+g)^{t_{Blue}}}{1+i} \right] - \frac{Green}{i-g} \cdot \left[ 1 - \frac{(1+g)^{t_{Green}}}{1+i} \right]
\]

**Final Design**

After all of these adjustments and redesign effort, the result is an optimization design based on the following calculations.

Decision variables:

\[ x_{ij} = \text{decision variable for category } i, \text{ alternative } j \]

Objective function:

Max: \( w_{\text{Savings}} \cdot Total_{\text{-Savings}} + w_{\text{ROI}} \cdot ROI + w_{\text{Payback}} \cdot Payback_{\text{-Period}} \)

Subject to:

- SINGLE-SYSTEM constraints
- PHYSICAL constraints
- CONVENIENCE constraints
- CLIENT-IMPOSED constraints

*(See Constraints list above)*

Equations:

TOTAL NPV of all investments:

\[
NPV = \sum_{i=H}^{i=A} S_i - \sum_{i=H}^{i=A} I_i + \sum_{i=H}^{i=A} DRC_i
\]
TOTAL SAVINGS for heating, cooling, hot water, refrigerator, clothes washer:

$$S_i = \sum_{j \in i} y_{ij} \cdot (y_{ij} - x_{ij}) \cdot (PV_{i,3} + PV_{i,2} - PV_{i,1})$$

$$PV_{i,1} = \frac{(oc_{i,existing} - oc_{i,HES})}{i - g} \cdot \left[ 1 - \left( \frac{1 + g}{1 + \bar{g}} \right)^{(l_{i,j} - age_{i,existing})} \right]$$

$$PV_{i,2} = \frac{(oc_{i,average} - oc_{i,HES})}{i - g} \cdot \left[ 1 - \left( \frac{1 + g}{1 + \bar{g}} \right)^{(l_{i,HES} - l_{i,existing} + age_{i,existing})} \right]$$

$$PV_{i,3} = \frac{(oc_{i,average} - oc_{i,HES})}{i - g} \cdot \left[ 1 - \left( \frac{1 + g}{1 + \bar{g}} \right)^{age_{i,existing}} \right]$$

$$oc_{existing} = \begin{cases} 
\text{eff_{i,input}} \cdot (1 - deg_{ij})^{age_{i,existing}} \\
\text{eff_{i,year}} \cdot (1 - deg_{ij})^{age_{i,existing}} 
\end{cases}$$

TOTAL SAVINGS for insulation, add-ons:

$$S_i = s_{ij} \cdot \left[ 1 - \left( \frac{1 + g}{1 + \bar{g}} \right)^{(l_{i,j})} \right]$$

TOTAL INVESTMENT for all categories:

$$I_i = \sum_{j \in i} y_{ij} \cdot (y_{ij} - x_{ij}) \cdot p_{ij}$$

TOTAL DRC for heating, cooling, hot water, refrigerator, clothes washer:

$$DRC_i = \sum_{j \in i} y_{ij} \cdot (y_{ij} - x_{ij}) \cdot \frac{p_{average}}{(1 + \bar{g})^{l_{i,j} - age_{i,existing}}}$$

Where

- $s_{ij}$ = yearly savings by selecting alternative ij
- $l_{ij}$ = expected lifespan of alternative ij
- $p_{ij}$ = purchase price of new system ij
- $oc_{ij}$ = operating cost of system ij
- $y_{ij}$ = existing system (binary) for category i, alternative j
- $\text{eff_{i,input}}$ = system efficiency of alternative i, as input by user
- $\text{eff_{i,year}}$ = system efficiency of alternative i, by installation year
- HES = new high-efficiency system
- average = average new system
- existing = system currently installed
Sample Run

To verify and demo the optimization program, an example home was used, in conjunction with reasonable expectations of budget and risk tolerance. The existing system, recommendations, and results are listed below.

<table>
<thead>
<tr>
<th>System</th>
<th>Existing</th>
<th>Recommended Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heating</td>
<td>Electric furnace</td>
<td>Geothermal</td>
</tr>
<tr>
<td>Cooling</td>
<td>Central air</td>
<td>Geothermal</td>
</tr>
<tr>
<td>Domestic hot water</td>
<td>Electric storage</td>
<td>Desuperheater</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>Top freezer</td>
<td></td>
</tr>
<tr>
<td>Clothes washer</td>
<td>Front load</td>
<td></td>
</tr>
<tr>
<td>Insulation</td>
<td>5” Fiberglass</td>
<td>Add 5” fiberglass</td>
</tr>
<tr>
<td>Add-on features</td>
<td></td>
<td>Install 4 ceiling fans</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Replace all incandescent bulbs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Install set-back thermostat</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use existing power strips for efficiency</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Purchase 1 additional power strip</td>
</tr>
</tbody>
</table>

Return on Investment = 198%
Total Savings = $35,468
Payback period = 13.4 years

Potential Extensions

There are several extensions that would make this optimization a more useful tool for the client. They fall into four categories: adjustability, additional features, and reporting.

Adjustability

Several assumptions that had to be made to estimate the energy use in the house being analyzed should be revisited. These include the square footage of the house (scaling relevant costs appropriately), the era the house was built in (which affects insulation options as well as likely existing conditions), and how the homeowner lives in the home (such as the temperature they keep the house at, which affects energy use significantly).
Additional features
Three categories of features were not addressed in this analysis, but still have significant impact on a house’s energy consumption: windows, air sealing, and electronics usage. These items were not included in this model because of their complexity and the difficulty of incorporating them; however, a comprehensive tool should factor them in, in some way.

Reporting
Currently, the simulation output is simply a string of decision variables, with a few cumulative statistics such as ROI. To make the output more useful, a graphical representation of yearly savings, as well as a comparison of the projected ROI with other investment options, should be built. Additionally, a formal report with a prioritized list of investments, based on the client’s preferences, should be included.

Conclusion
This project has been successful, as it has resulted in a useful, portable tool that can be used by Company B to enhance its energy audit service. The optimization uses client-specific input, combined with researched assumptions, to provide detailed estimates of savings, return on investment, and payback periods. This information is used to prioritize investment strategies and results in a list of recommendations for the client.
## Appendix

**Exhibit 1: All alternatives included in the model.**

<table>
<thead>
<tr>
<th>Heating</th>
<th>Cooling</th>
<th>Domestic Hot Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Furnace (natural gas)</td>
<td>Window units</td>
<td>Standard (electric)</td>
</tr>
<tr>
<td>Furnace (electric)</td>
<td>Central air (electric)</td>
<td>Standard (natural gas)</td>
</tr>
<tr>
<td>Boiler (natural gas)</td>
<td>Central air (geothermal)</td>
<td>Desuperheater (geothermal)</td>
</tr>
<tr>
<td>Central air (geothermal)</td>
<td></td>
<td>Instant-on (electric)</td>
</tr>
<tr>
<td>Radiator (electric)</td>
<td></td>
<td>Instant-on (natural gas)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Refrigerator</th>
<th>Stove</th>
<th>Clothes Washer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Freezer</td>
<td>Glass top (electric)</td>
<td>Front-load</td>
</tr>
<tr>
<td>Bottom Freezer</td>
<td>Standard (electric)</td>
<td>Top-load</td>
</tr>
<tr>
<td>Side-by-side</td>
<td>Standard (natural gas)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Insulation</th>
<th>Add-ons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiberglass</td>
<td>Ceiling fans</td>
</tr>
<tr>
<td>Cellulose</td>
<td>CFL replacement</td>
</tr>
<tr>
<td>Foam</td>
<td>Set-back thermostat</td>
</tr>
<tr>
<td></td>
<td>Power strips (use)</td>
</tr>
<tr>
<td></td>
<td>Power strips (buy)</td>
</tr>
<tr>
<td></td>
<td>Caulk</td>
</tr>
<tr>
<td></td>
<td>Hot water blanket</td>
</tr>
<tr>
<td></td>
<td>Ducting insulation</td>
</tr>
</tbody>
</table>
Exhibit 2: Using Energy-10. Assumptions listed are conservative estimates for a standard St. Louis home, and are based on St. Louis building codes. Also included below are sample results.

<table>
<thead>
<tr>
<th>Building characteristic</th>
<th>Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>Two story; 2,000 square feet; aspect ratio 1.5</td>
</tr>
<tr>
<td>Wall construction</td>
<td>2x4 frame; 6&quot; fiberglass insulation; R-12.6</td>
</tr>
<tr>
<td>Roof construction</td>
<td>Flat, R-19</td>
</tr>
<tr>
<td>Floor construction</td>
<td>Slab on grade; Reff=8.6</td>
</tr>
<tr>
<td>Windows</td>
<td>4x6; wood framing; low-e coating; U=.47; 6 on N/S facades; 4 on E/W facades.</td>
</tr>
<tr>
<td>Internal gains</td>
<td>Internal lights – 2.0 W/sqft; External lights -.4 W/sqft; People – 4; Hot water -.66 W/sqft; Other -.36 W/sqft</td>
</tr>
<tr>
<td>HVAC auto-sizing</td>
<td>Oversized by 20%</td>
</tr>
</tbody>
</table>

Comparing the energy usage between two HVAC systems, with all other assumptions the same. Red – Gas furnace with 70% efficiency. Green – Gas furnace with 80% efficiency.